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# A NONLINEAR ANALYSIS METHOD FOR ARCH-SHAPED SHELL ROOFS MADE OF COLD-FORMED STEEL PROFILES

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**Abstract:** The paper deals with the problem of static structural analysis of shallow arch-shaped shell roofs assembled from curved cold-formed steel profiles. A nonlinear analysis procedure has been proposed to estimate the stress-strain state of such roof structures. It is based on two interrelated analysis models developed for the purpose – namely, an action effects model used for the static analysis of deformational behaviour of shallow arch-shaped structural systems, and a resistance model, for strength analysis of cold-formed steel profiles.

*Keywords*: arch-shaped shell roof, cold-formed steel profiles, nonlinear analysis method, action effects model, structural resistance model.

## 1. Introduction

The arch-shaped shell roof made of curved cold-formed steel profiles is a lightweight frameless roofing structure designed to perform both a load-bearing and enclosing function. It is said to be shallow if its rise-to-span ratio does not exceed 1/8 in value. Despite all the research and experimental work that has been done to date, the problem of modelling and analysing the behaviour of such structures still persists since it turns out to be rather complex both in theoretical and practical aspects.

Basically, there are three different types of structural models which can be used for the stress-strain analysis of arch-shaped shell roofs: a 3-D folded shell model (Biegus and Kowal, 2013, Casariego et al., 2011, Narvydas and Puodžiūnienė, 2013, Zahurul Islam et al., 2005), a 3-D thin shell model (Abdel-Sayed et al., 1980, El-Atrouzy, 1997, Krasotina, 2014), and a 2-D curved bar model (Abdel-Sayed et al., 1980, La Puebla-Ferri et al., 2009, Makelainen and Hyvarinen, 1990). Unfortunately, none of these structural models permits one to carry out such an analysis and interpret its results both effectively and efficiently, for the time being at least.

Thus, the 3-D folded shell model, while allowing the strain-and-stress state to be evaluated with utmost precision at any given point of the shell, turns out to be hugely time-and labour consuming and costly to construct.

The thin shell model, in contrast, takes one less time and effort to create, but in order to provide accurate solutions, first, the shell needs to be orthotropic, and secondly, provisions should be made in the analysis method to take into account the non-linear peculiarities of thin-walled structures' behaviour under loading, which is not an easy thing to do.

Finally, the curved bar model, though being the least complex one, can be used only in the case of arch-shaped shell roof structures with both uniform boundary conditions and loads uniformly distributed in direction perpendicular to the roof's span. Moreover, in order to provide realistic solutions it has to take into account possible structural effects due to geometrical non-linearity as well as local and distortional buckling.

The paper describes a novel nonlinear strain-stress analysis procedure developed by the authors on the basis of the 2-D curved bar model which takes into account the above mentioned structural effects.

## 2. Nonlinear analysis method and its constituents

The nonlinear analysis procedure presented in this paper is based on two functionally interrelated analysis models – namely, an action effects model and a structural resistance model which correspondingly allow for geometric nonlinearity ($2^{nd}$ order effects) and both local and distortional buckling to be taken into account.

The action effects model is based on the theory of deformational analysis of both arched and cable-stayed structural systems (Ulasevich, 1984). The span part of such systems is viewed as an elastic curved bar fixed on elastic supports, and being in equilibrium under the action of a specified initial state load $g_0(x)$ (Fig. 1).

As to the initial state itself, it is assumed to be given

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by an adequate number of parameters and their values so that all the other ones that are dependent on them can be calculated on the basis of equilibrium equations only. All equilibrium states other than the initial one are taken as the deformed ones, for all of which, with the exception of equilibrium equations, it is necessary to consider linear and angular deformation equations when calculating the required parameters values in the general case.

The equilibrium equation for a bar in the deformed state is of the form:

$$\frac{d^4 v}{dx^4} - \frac{H}{EI} \cdot \frac{d^2 v}{dx^2} = \frac{H - H_0}{EI} \cdot \frac{d^2 y_0}{dx^2} - \frac{g(x)}{EI} \tag{1}$$

where: $EI$ is the bending stiffness of the elastic bar in the deformed state; $H_0$ and $H$ are the arch thrust in the initial and deformed equilibrium states respectively; $y_0$ is the curved bar axis shape in the initially deformed state, taken in (1) as

$$y_0 = \frac{1}{H_0}\left[\left(\sum_{k=1}^{n} G_k^0 (l - t_k)\right)\frac{x}{l} - \sum_{k=1}^{m_x} G_k^0 (x - t_k) - M_0\right] + x \cdot tg\varphi \tag{2}$$

with $0 \le x \le l$ and $0 \le t_k \le x$.

The bending moment $M_0$ in (2) equals

$$M_0 = \frac{r_0}{a_0} \cdot \sin(a_0 \cdot x) - \frac{1}{a_0} \sum_{k=1}^{m_x} G_k^0 \cdot \sin[a_0 (x - t_k)] \tag{3}$$

where:

$$a_0 = \sqrt{\frac{|H_o|}{EI_0}}$$

$$r_0 = \frac{1}{\sin(a_0 \cdot l)} \sum_{k=1}^{n} G_k^0 \cdot \sin[a_0 (l - t_k)]$$

and $EI_0$ is the bending stiffness of an elastic bar in the initial state.

The general solution of equation (1) obtained in the analytical form, with the arbitrary load function $g(x)$ replaced by a system of discrete forces $G_k$ on the basis of the mean value theorem, can be written as follows:

$$v = \frac{1}{H}\left[\left(\sum_{k=1}^{n} P_k (l - t_k)\right)\frac{x}{l} - \sum_{k=1}^{m_x} P_k (x - t_k) + M^a - \left(M^a - M^b\right)\frac{x}{l}\right] - \bar{y}_0 \tag{4}$$

where: $P_k = G_k^0 + G_k$; $\bar{y}_0 = y_0 - x \cdot \mathrm{tg}\varphi$; $M^a$ and $M^b$ are the support moments.

The bending moment $M$ in (4) equals

$$M = \frac{r}{a}\sin(a \cdot x) - \frac{1}{a}\sum_{k=1}^{m_x}\left(P_k - \beta G_k^0\right)\sin[a(x - t_k)] + \frac{M^a}{\sin(a \cdot l)}\sin[a(l - x)] + \frac{M^B}{\sin(a \cdot l)}\sin(a \cdot x) + \beta M_0 \tag{5}$$

where:

$$a = \sqrt{\frac{|H|}{EI}},$$

$$\beta = \frac{EI - EI_0}{(EI \cdot H_0)/(H - EI_0)},$$

$$r = \frac{1}{\sin(a \cdot l)} \sum_{k=1}^{n}\left(P_k - \beta G_k^0\right) \cdot \sin[a(l - t_k)].$$

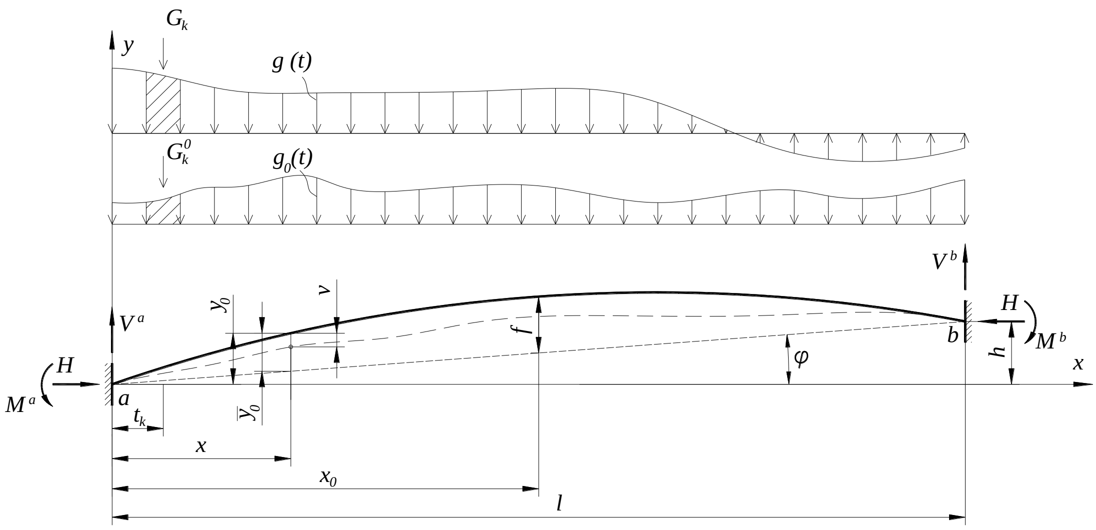

Fig. 1. Analysis model for elastic curved bar.

Function (4) under longitudinal and angular deformation conditions makes it possible to form a non-linear system of basic equations to be solved for $H$, $M^a$ and $M^b$:

$$\left.\begin{aligned}&\frac{H\cdot l}{EA\cdot\cos^2(\varphi)}+\left(\frac{H}{EA}-\frac{\cos^3(\varphi)}{2}\right)\frac{D(H)}{H^2}+\frac{1}{\cos(\varphi)}\cdot\\&\cdot\left[(H-H_0)\left(c_h^a-c_h^b\right)+\left(\alpha\cdot l\cdot\Delta t-\frac{P_n\cdot l}{EA}\right)\right]+J_0=0\\&\left(\frac{tg(a\cdot l)-a\cdot l}{H\cdot l\cdot tg(a\cdot l)}-c_y^a\left[1+\left(\frac{dy_0}{dx}\right)^2_{|x=0}\right]\right)M^a-\\&-\frac{\sin(a\cdot l)-a\cdot l}{H\cdot l\cdot\sin(a\cdot l)}M^b=\frac{dv}{dx}\Big|_{x=0,M^a=0,M^b=0}\\&\frac{\sin(a\cdot l)-a\cdot l}{H\cdot l\cdot\sin(a\cdot l)}M^a-\left(\frac{tg(a\cdot l)-a\cdot l}{H\cdot l\cdot tg(a\cdot l)}-\right.\\&\left.-c_y^b\left[1+\left(\frac{dy_0}{dx}\right)^2_{|x=l}\right]\right)M^b=\frac{dv}{dx}\Big|_{x=l,M^a=0,M^b=0}\end{aligned}\right\}\qquad(6)$$

where:

$$J_0=\frac{1}{H_0^2}\left(\frac{\cos^3\varphi}{2}-\frac{H_0}{EA_0}\right)\cdot\int_0^l\left(\frac{1}{l}\sum_{k=1}^{n}G_{0,k}(l-t_k)-\sum_{k=1}^{n}G_{0,k}-\frac{dM_0}{dx}\right)^2dx-\frac{H_0\cdot l}{EA_0\cos^2\varphi}$$

$$D(H)=\int_0^l\left(\frac{x}{l}\sum_{k=1}^{n}P_k\,(l-t_k)-\sum_{k=1}^{n}P_k-\frac{1}{l}\left(M^a-M^b\right)-\frac{dM}{dx}\right)^2dx$$

α is the linear temperature expansion coefficient; $\Delta t$ is the change in temperature with relation to initial state; $P_n$ is the prestressing force; $c_y$ and $c_h$ are the angular and linear elastic characteristics of arch supports; $EA_0$ and $EA$ is the arch longitudinal stiffness at the initial and deformed equilibrium states respectively.

Expressions (2), (3), (5) and (6) are valid only for arched structural systems functioning like a compressed-bent bar ($H / EI < 0$), while in the case of their functioning like a tensioned-bent bar ($H / EI > 0$), the trigonometric functions should be replaced by the hyperbolic ones.

Peculiarities of the structural behaviour of thin-walled cold-formed steel profiles due to local and distortional buckling are taken into consideration in the structural resistance model by way of using the so-called effective section properties, instead of the gross ones.

The effective cross-section properties are normally determined following the procedures specified in EN 1993-1-3 & EN 1993-1-5. Thus, the local buckling of plane elements without intermediate stiffeners is taken into account by reducing the gross width of plane elements, while flexural buckling of the stiffener is done by reducing the thickness of both the stiffener itself and the adjacent effective portions of plane elements.

However, the basic analysis procedure specified in EN 1993-1-3 doesn't bring out the real stress-strain state of cold-formed steel profiles as it requires that the effective cross-section properties be calculated at the values of stresses in the compressed elements corresponding to the yield strength of steel (Zhdanov and Ulasevitch, 2013). Furthermore, in accordance with EN 1993-1-3 rules, the effective area $A_{eff}$ should be determined assuming that the cross section is subject only to stresses due to uniform axial compression, while the effective moment of inertia $I_{eff}$ should be calculated assuming that the cross section is subject only to bending stresses (Ulasevitch and Zhdanov, 2015), so the calculated effective cross-section properties can significantly differ from the real ones.

In order to obtain more realistic values of the effective properties, an analysis procedure that uses the real stress distribution under a simultaneous action of axial force NEd and bending moment $M_{Ed}$, acting simultaneously, has been developed (Fig. 2).

The effective width of the compressed plane elements is calculated on the basis of the buckling curve as given in EN1993-1-5:

$$\rho=\begin{cases}1{,}0\\ \bar{\lambda}_p-\dfrac{0{,}055(3+\psi)}{\bar{\lambda}_p^2}\le 1{,}0\end{cases}\qquad(7)$$

for

$$\bar{\lambda}_p\le 0{,}5+\sqrt{0{,}085-0{,}055\psi}$$

where $\psi$ is the stress ratio.

The relative slenderness in (7) is taken as

$$\bar{\lambda}_p=\frac{\bar{b}}{t}\cdot\frac{1}{\sqrt{k_\sigma}}\cdot\sqrt{\frac{\sigma_{com.Ed}\cdot\gamma_{M0}}{E}\cdot\frac{12(1-\mu)}{\pi^2}}=\frac{\bar{b}}{t}\cdot\frac{0{,}951}{\sqrt{k_\sigma}}\cdot\sqrt{\frac{\sigma_{com.Ed}\cdot\gamma_{M0}}{E}}\qquad(8)$$

where: $\bar{b}$ is the appropriate flat width; $t$ is the thickness of compressed element; $k_\sigma$ is the buckling factor corresponding to the stress ratio $\psi$ and boundary conditions; $E$ is the modulus of elasticity of steel; $\mu$ is the Poisson's ratio of steel; $\sigma_{com.Ed}$ is the maximum design compressive stress in the element caused by all simultaneous actions; $\gamma_{M0}$ is the partial safety factor.

To calculate the reduced thickness of intermediate stiffeners in compression use is made of the buckling curve as given in EN1993-1-3:

$$\chi_d=\begin{cases}1{,}0 & \bar{\lambda}_p\le 0{,}65\\ 1{,}47-0{,}723\cdot\bar{\lambda}_d & \text{for } 0{,}65\le\bar{\lambda}_p\le 1{,}38\\ 0{,}66/\bar{\lambda}_d & \bar{\lambda}_p\ge 1{,}38\end{cases}\qquad(9)$$

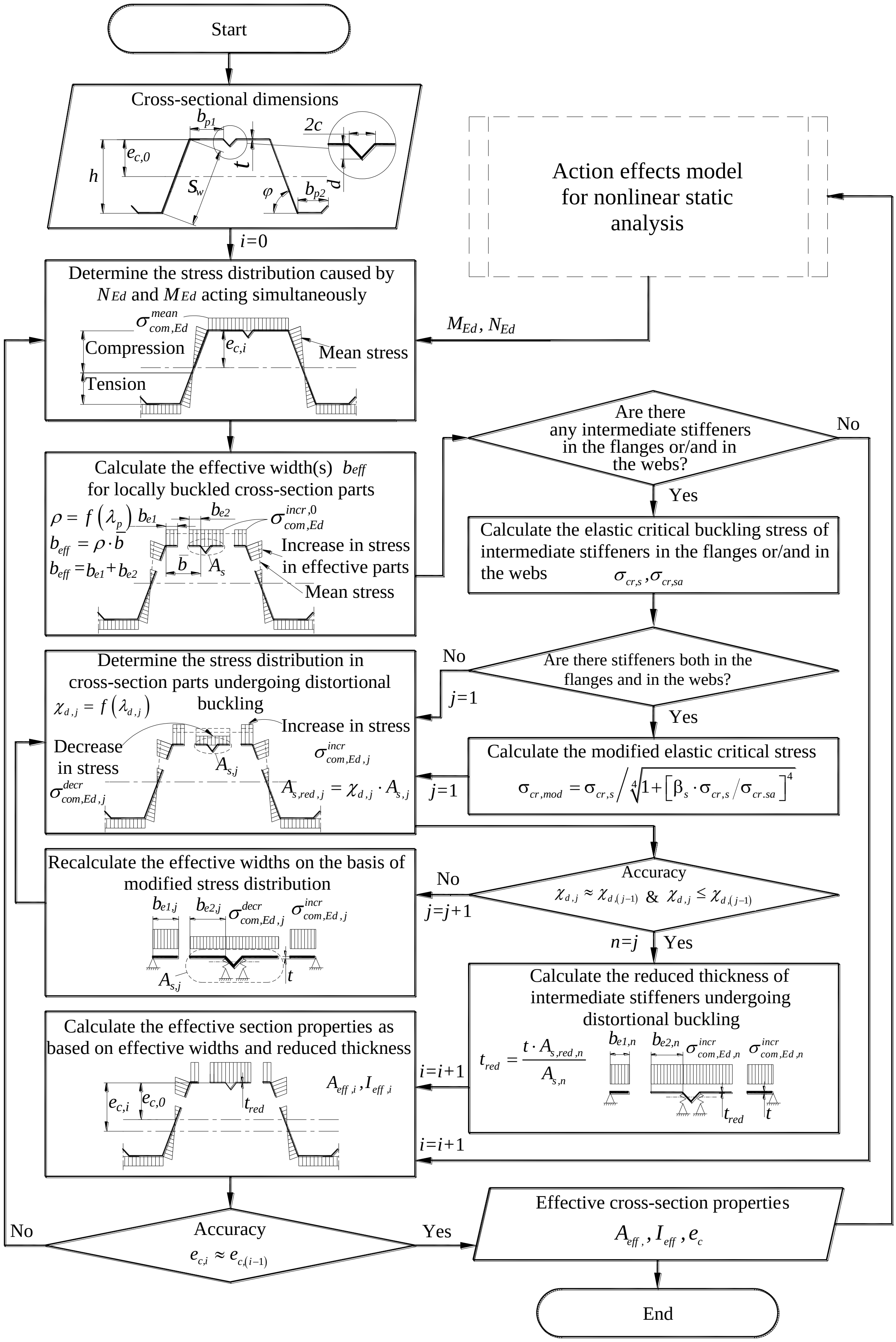


Fig. 2. Flow-chart for determining effective section properties.

The relative slenderness in (9) is taken as

$$\bar{\lambda}_d = \sqrt{\frac{\sigma_{com.Ed} \cdot \gamma_{M0}}{\sigma_{cr,s}}} = \qquad (10)$$

where: $\sigma_{cr.s}$ is the elastic critical stress for a stiffener.

Using effective section properties $A_{eff}$, $I_{eff}$ and $e_c$, determined through the proposed procedure, the load bearing capacity of the curved cold-formed profiles forming a shell roof structure can be checked as follows:

$$\frac{N_{E,d}}{N_{c,Rd}} + \frac{M_{y,Ed+\Delta M_{y,ED}}}{M_{cy,Rd,com}} \leq 1 \qquad (11)$$

where: $N_{c,Rd} = A_{eff} \cdot f_{yb} / \gamma_{M0}$ is the design resistance of the cross-section to compression; $M_{cy,Rd,com} = W_{eff} \cdot f_{yb} / \gamma_{M0}$ is the design moment resistance of the cross-section to bending; $\Delta M_{y,Ed} = N_{Ed} \cdot e_{Ny}$ is the additional moment due to the shift of the centroidal axis; $e_{Ny}$ is the shift of *y-y* centroidal axis due to axial forces.

## 3. Conclusion

There are a number of structural analysis methods a structural engineer or researcher can make use of to predict the behaviour of arch-shaped shell roofs made of cold-formed steel profiles. However, the problem is that the analysis procedures that are simple and easy enough to perform, turn out to lack accuracy, while those that prove to be highly accurate, are far too much time- and labour consuming.

In this paper, a novel nonlinear strain-stress analysis method has been proposed. It is based on two functionally interrelated analysis models – namely, an action effects model and a structural resistance model which correspondingly allow for geometric nonlinearity as well as local and distortional buckling to be taken into account. When implemented in a corresponding computer program, the technique is likely to significantly reduce the time and effort currently needed to effectively and efficiently analyse the arch-shaped shell roofs made of curved cold-formed steel profiles, with the required level of reliability at that.

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